Reflected Power Effects in Computer Simulations Using the Quantum Theory of Mixing

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ABSTRACT

Attention is directed to the signal reflection gain and the signal-to-image conversion gain in the quantum theory of mixing. The theory gives two distinct types of solutions for the minimum noise temperature of an SIS receiver. One has very high IF conversion gain, and the returned signal and image powers are extremely high as well. The other has moderate IF conversion gain, but the returned powers tend to be very small. This resolves a longstanding mystery.

INTRODUCTION

Classical mixer theory does not apply to heterodyne mixers operating at a frequency ω high enough that the voltage scale of the resistive nonlinearity is comparable to or smaller than $\hbar\omega/e$. Rather, the quantum theory of mixing [1] developed by J.R. Tucker must be used. Tucker's theory is very successful in predicting the behavior of superconductor-insulator-superconductor (SIS) quasiparticle mixers, the most sensitive receiving devices in the vicinity of 100 to 300 GHz. We have attempted to use the theory to determine the best performance which can be expected from SIS receivers, with reasonable but optimistic experimental constraints, over their entire frequency range of operation. This project is not yet complete, but we report here one striking result which should be considered by other researchers who attempt to use the Tucker theory to simulate their experimental results.

We find there are two distinct and disjoint types of "optimum" solutions. One has very high or infinite IF conversion gain, and the "reflected" power (the signal reflection gain and the signal-to-image conversion gain) is very high as well. The other has moderate IF conversion gain, but the reflected power tends to be very small. For a wide range of experimental parameters the high gain solution gives the lowest receiver noise temperature. Nevertheless, the high gain solution is likely not accessible in real experiments. Therefore, eliminating the high gain solution should give much better agreement with experiments.

CALCULATIONS

We have used the quantum mixer theory for extensive numerical calculations, to determine the minimum value of the SSB (single sideband) receiver noise temperature at each frequency, subject to reasonable experimental constraints. Because the noise temperature of a receiver is

$$T_R = T_M + LT_{IF}, \tag{1}$$

the calculation involves a trade-off between minimizing the mixer noise temperature, T_M, and minimizing the mixer conversion loss, L, which is mediated by the noise temperature of the IF amplifier, T_{IF}. Full details of these calculations will appear elsewhere. For our current purpose we make the following approximations: We consider DSB (double sideband) operation in the three-frequency low-IF approximation, which should be a fairly good representation of most well-designed experimental mixers. We do not include any interference from the Josephson effect, although this is likely to be a problem for experiments at the higher frequencies. In addition, we ignore all reactances. Taken together, these approximations are equivalent to assuming 1) that the geometrical capacitance of the SIS junction is large enough to both short out the LO harmonics and their sidebands and to eliminate Josephson interference, 2) that the capacitance is itself resonated by a relatively broadband external tuning circuit, so that the intrinsic junction nonlinearity is presented with a resistive embedding impedance at all relevant frequencies, and 3) that the quantum susceptance has no significant effect. This third assumption is controversial. It has recently been argued that the quantum susceptance is a central element of the behavior of SIS mixers [2]. Nevertheless, we believe that this nonlinear reactance has little effect on the performance of an optimized SIS receiver, although it may affect the optimum bias point. This question will be addressed in further research.

The equations employed in the calculation of T_R are taken from Ref. [1]. For brevity only those discussed in this paper are presented here. The SSB IF conversion loss (inverse conversion gain) of a DSB SIS mixer in the three-frequency low-IF approximation, ignoring reactances, can be written in the novel form:

MM

$$L \equiv \mathcal{G}_{IF}^{-1} = \frac{(G_{00}+G_L)^2}{G_L G_{01}^2}\,\frac{(G_s+G_s')^2}{4G_s}\,, \tag{2}$$

$$G_s' \equiv G_{11} + G_{1\text{-}1} - \frac{2G_{01}G_{10}}{G_{00}+G_L}\,. \tag{3}$$

L depends upon G_s, the source conductance seen by the SIS junction at the signal and the image frequencies, and upon the load conductance G_L which represents the conductance of the IF amplifier circuitry as seen by the junction. The dependence of L upon G_s in Eq. 2 is given by a simple impedance matching formula which has its minimum at $G_s = |G_s'|$. The quantity G_s', which can be negative, is equal to the input conductance of a mixer for the unusual case of a DSB signal resulting from an amplitude modulation of the LO. Note that G_s' is not the input conductance of our mixer. (The simple amplitude modulation model of the SIS mixer is described in Ref. [1], Sec. III.B.) We will also refer to the input conductance of the mixer at the local oscillator frequency ω,

$$G_{LO}^{\circ} \equiv I_{LO}/V_{LO} = G_{11} - G_{1\text{-}1}\,, \tag{4}$$

where I_{LO} and V_{LO} are the amplitudes of the current and voltage across the junction at frequency ω. Of course $G_{LO}^{\circ} > 0$.

Equations 2 - 4 are expressed in terms of the elements of the small-signal conductance matrix, G_{ij}. Each G_{ij} is evaluated as an infinite sum over index n of the currents $I_n \equiv I_{dc}(V_n)$ weighted by a combination of Bessel functions of argument $\alpha = eV_{LO}/\hbar\omega$, where $I_{dc}(V)$ is the unmodulated dc I-V characteristic of the SIS junction, $V_n \equiv V_0 + n\hbar\omega/e$, and V_0 is the dc bias voltage.

The mixer noise temperature T_M includes the shot noise calculated according to [1] and the thermal noise from the image termination. The latter does not depend upon any of the operating parameters of a DSB mixer and hence does not influence the receiver optimization. For convenience we assume zero physical temperature so the thermal noise simply reduces to the quantum noise temperature of the mixer, $\hbar\omega/2k$. In this paper we ignore the thermal noise from the IF termination which is reflected from the mixer back into the IF amplifier. For real SIS receivers this can be an important contribution to the total noise.

We pay close attention to the reflected power. It is known that a good quality SIS junction can give infinite gain at frequencies up to twice the effective energy gap frequency. Although this might seem advantageous in the context of Eq. 1, when the IF conversion gain is infinite the output power at all sideband frequencies is infinite as well [3], clearly an unstable situation. Reference [4] shows that SIS mixers can be unstable in a wide range of circumstances. For our mixer the signal reflection gain and the signal-to-image conversion gain are respectively:

$$\mathcal{G}_S = \frac{1}{4}\Big(\frac{G_s - G_s'}{G_s + G_s'} + \frac{G_s - G_{LO}^{\circ}}{G_s + G_{LO}^{\circ}}\Big)^2, \tag{5}$$

$$\mathcal{G}_I = \frac{1}{4}\Big(\frac{G_s - G_s'}{G_s + G_s'} - \frac{G_s - G_{LO}^{\circ}}{G_s + G_{LO}^{\circ}}\Big)^2. \tag{6}$$

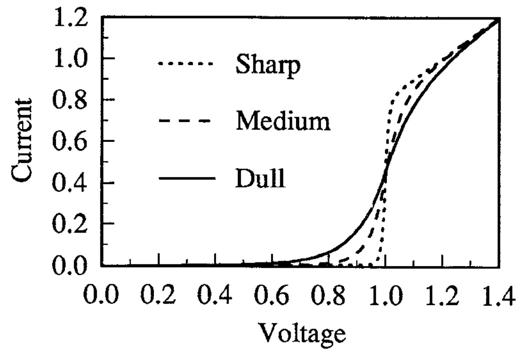

Fig. 1. Three synthetic normalized I-V characteristics used for these calculations.

There has been little appreciation of $\mathcal{G}_S$ and $\mathcal{G}_I$ in the literature. An exception is Ref. [5], which enforced an approximate signal input impedance match. But to our knowledge the present paper is the first to mention the signal-to-image conversion gain of an SIS mixer in any context.

We use the equations of the quantum theory of mixing to determine the minimum noise temperature of the SIS receiver at each frequency. At each frequency the optimum values of G_s, V_0, and α are calculated given discrete values for the remaining parameters. We have performed these calculations for a wide range of parameters, but only a few of the results can be presented here. The illustrations given in this paper use the synthetic SIS junction I-V curves depicted in Fig. 1. Most of our results pertain to the "sharp" curve, corresponding to the best experimental SIS I-V curves; the "medium" curve corresponds to a good quality junction and the "dull" curve corresponds to a moderate quality junction. We normalize voltages to the energy gap voltage V_g, conductances to the normal state resistance R_N, and frequencies to the energy gap frequency $\omega_g \equiv eV_g/\hbar$.

RESULTS

The dotted curve of Fig. 2 shows the minimum theoretical noise temperature of an SIS receiver using the "sharp" I-V curve, with $G_L = 0.3/R_N$, $T_{IF} = 3$ K, and $V_g = 3$ mV, for frequencies up to $\omega = 0.2\ \omega_g$. The smoothness of this curve hides the fact that it includes two distinct and disjoint types of behavior. The thick lines in Fig. 3 represent the three corresponding small-signal gains. It is seen that $\mathcal{G}_{IF}$ is extremely high, and $\mathcal{G}_S$ and $\mathcal{G}_I$ are even higher, for normalized frequencies between 0.03 and 0.12. For other frequencies $\mathcal{G}_{IF}$ is more moderate but still sizable, while $\mathcal{G}_S$ and $\mathcal{G}_I$ are quite small.

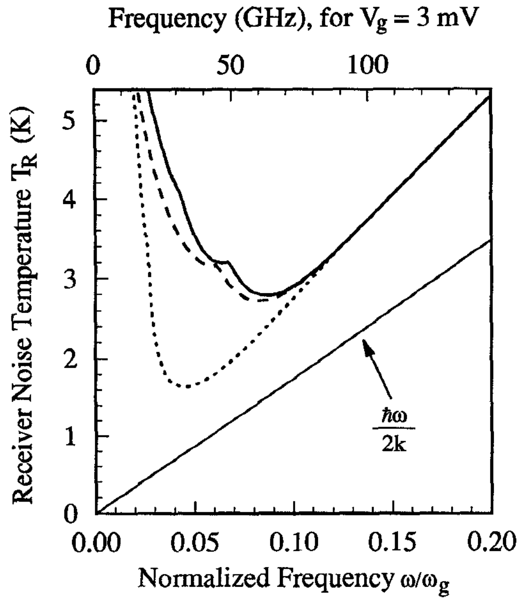

Fig. 2. The SSB noise temperature of a DSB SIS receiver optimized at each frequency. The computation uses the "sharp" I-V curve of Fig. 1, $G_L = 0.3/R_N$, T_{IF} = 3 K, and V_g = 3 mV. The dotted curve is the universal minimum of T_R, the dashed curve is calculated with the constraint $G_S' > 0$, and the solid curve is calculated with the constraint $\mathcal{G}_S$, $\mathcal{G}_I \leq 1/4$.

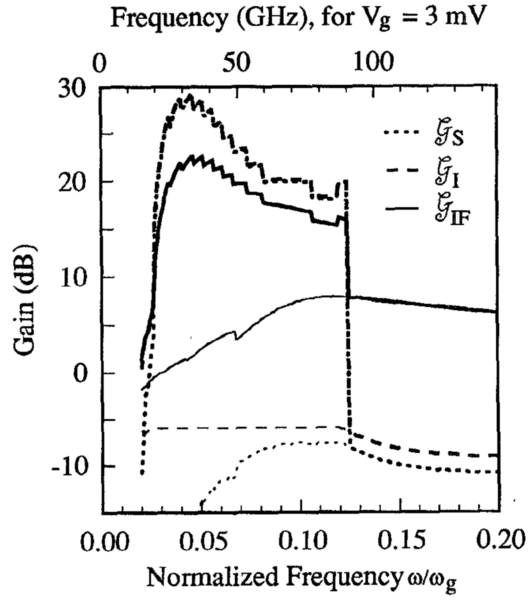

Fig. 3. The thick lines represent the IF conversion gain $\mathcal{G}_{IF}$, the signal reflection gain $\mathcal{G}_S$, and the signal-to-image conversion gain $\mathcal{G}_I$, which correspond to the dotted curve of Fig. 2. The thin lines represent the gains which correspond to the solid curve of Fig. 2.

The solid curve of Fig. 2 shows the minimum theoretical noise temperature of the SIS receiver calculated using exactly the same parameters as the dotted curve, but subject to the constraint $\mathcal{G}_S$, $\mathcal{G}_I \leq 1/4$, corresponding to a VSWR of 3. The thin lines in Fig. 3 represent the three corresponding small-signal gains. It is clear that the reflected power constraint eliminates the high gain solution and allows the moderate gain solution to extend continuously across the entire frequency region.

This illustrates a general feature of our results: For a wide range of parameter values there are two distinct minima for T_R. One has very high or infinite IF conversion gain, and the reflected power is very high or infinite as well. The other has moderate IF conversion gain, but the reflected power tends to be very small. Our minimization routine selects the lower of the two minima. (Frequently the two minima overlap in parameter space, but this has no substantial effect on our argument.)

Another general feature of our results is that if care is taken to eliminate the high gain solutions, the optimum T_R is then quite insensitive to the level of reflected power allowed. Figure 2 illustrates this. The dashed curve is computed with the requirement $G_S' > 0$ (the high gain solutions generally but not always have $G_S' < 0$). Even though for this calculation $\mathcal{G}_I$ (not shown) reaches as large as 0.78, Fig. 2 shows that the optimum T_R is nearly the same as the solid curve for which $\mathcal{G}_I \leq 0.25$. In fact, for the parameters used for Figs. 2 and 3, *any* reasonable constraint on the maximum allowed $\mathcal{G}_S$ and $\mathcal{G}_I$, from 0.1 to 10, serves to eliminate the high gain solutions but has very little effect on the moderate gain solutions.

Although we have used particular parameter values in Figs. 2 and 3 for illustration, the behavior described is quite widespread. For instance, for the "medium" I-V curve of Fig. 1 with T_{IF} = 10 K, the high gain solution predominates for frequencies between $\omega = 0.15\ \omega_g$ and $\omega = 0.40\ \omega_g$.

DISCUSSION

We understand our results in the following way. T_M and L in Eq. 1 are in general slowly varying functions of their various parameters. The exception is that if it is possible for G_S' to be negative the quantity G_S+G_S' can go to zero, giving analytically infinite $\mathcal{G}_{IF}$ in Eq. 2. Near exact cancellation occurs over only a small region of parameter space. Thus we can picture a display of T_R in a multi-parameter space as having a rather sharp minimum near $G_S+G_S' = 0$ and more gradual behavior, including a broad minimum, elsewhere. These two minima correspond to our two solutions. Near $G_S+G_S' = 0$ both $\mathcal{G}_S$ and $\mathcal{G}_I$ will be very large (cf. Eqs. 5 and 6). If the vicinity of $G_S+G_S' = 0$ is forbidden (by constraining $\mathcal{G}_S$ and $\mathcal{G}_I$), the remaining minimum will be quite insensitive to the operating parameters.

This analysis supports our belief that, were we to include the quantum susceptance and a tuning susceptance in our calculations, the result would be very much the same. Including these reactive terms, a quantity exactly analogous to G_S+G_S' (denoted D in Ref. [3]) can go to zero giving analytically infinite $\mathcal{G}_{IF}$ as well as infinite $\mathcal{G}_S$ and $\mathcal{G}_I$ [3], and thus the above paragraph still applies. It is known that the quantum susceptance widens the range of possibility of infinite gain in SIS mixers [2], so that infinite gain can be predicted for instance for poorer quality I-V curves and over a wider frequency range. This should underline the importance of removing the high gain solutions in computer simulations.

Note in Fig. 3 that $\mathcal{G}_I$ is everywhere larger than $\mathcal{G}_S$. This is true for all of our simulations: for an optimized SIS receiver there is always more power returned to the source at the image frequency than at the signal frequency! We do not know the physical reason for this. Mathematically, it is clear from inspection of Eqs. 5 and 6 that this is a simple consequence of the fact that the optimum G_S is always intermediate between G_{LO}^{o} and G_S'. In fact $\mathcal{G}_S$ can be extremely small while $\mathcal{G}_I$ is still sizable, as at low frequencies in Fig. 3. Thus it is crucial to consider $\mathcal{G}_I$ as well as $\mathcal{G}_S$.

Our simulations make it clear that the high gain solutions should not be experimentally accessible. The very large returned power would make the mixer extraordinarily sensitive to small variations in the signal and image terminations. The high output power (at higher sideband frequencies as well, unless they are perfectly terminated) is extremely conducive to mixer saturation. The sensitivity of the high gain solutions to mixer operating parameters implies that these solutions will be obliterated by noise and other processes. It is likely that the high gain solution is a mere mathematical curiosity, forgoing a direct experimental assault.

We believe that this resolves a mystery. From the earliest attempts [6], the best experimental SIS mixers have had low to moderate conversion gain, but *optimized* computer fits to these experiments often predict very high to infinite gain. On the contrary, nonoptimized fits using independently determined parameters can be fairly successful [7,8]. Since the high gain solutions are not accessible, constraining the reflected power in computer fitting should give much better agreement with experiments.

CONCLUSION

It is important to consider the signal reflection gain and especially the signal-to-image conversion gain in computer simulations of SIS mixers. The computed minimum receiver noise temperature often entails extremely high signal and image returned power, and this type of solution is likely inaccessible in real experiments. If the high gain solutions are eliminated, it is possible to have very low returned power with little increase in noise temperature.

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